transition are retrieved when the offsets are zero. However, a large number of modes is required to obtain precise values.

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Moment Method Formulation of Thick Diaphragms in a Rectangular Waveguide

Amlan Datta, B. N. Das, and Ajoy Chakraborty

Abstract—**The paper presents a method of determination of the electrical characteristics of two thick apertures in a rectangular waveguide. The coupled integral equations resulting from the boundary condition of the magnetic field at the four interfaces are transformed into matrix equations using method of moments. The numerical data on reflection and transmission coefficients are evaluated. Comparison between theoretical and experimental results is presented.**

I. Introduction

The analysis of waveguide discontinuities in the form of thin and thick apertures has been carried out by a number of workers [1]–[4]. Marcuvitz used variational formulation for determining the equivalent network parameters of diaphragms with zero axial thickness and supplied experimental data on complex reflection coefficient for a diaphragm of thickness 0.08 cm in a rectangular waveguide [1]. The variational formulation was also applied to cylindrical posts with small circular and rectangular cross-section. The reference plane for lumped equivalent network representation of this structure was taken as the plane of symmetry of the obstacle. The application of this form is limited to obstacles having maximum linear dimension less than 10% of the waveguide broad dimension and for location with minimum distance of 30% of the guide broad dimension from the side wall. The analysis of apertures with finite axial thickness has also been carried out by Marcuvitz using the static method [1, Sect. 8.7–8.8]. The results are accurate for axial thickness much greater than the aperture width. In this case, however, the reference plane for lumped equivalent network representation has not been properly indicated presumably because of application of the static method. Collin has suggested a method for determination of the parameters of the equivalent T network of an inductive diaphragm with finite axial thickness [2]. The analysis is based on evaluation of the eigenvalues of the impedance matrix of the T network. In addition to some confusion about the reference plane for the network representation, the authors didn't find the method convenient for computation. The moment method formulation has recently been applied by Scharstein et al. [3] and Auda and Harrington [4] for thin diaphragms in circular and rectangular waveguides. The analysis of Scharstein et al. for an iris in a circular waveguide is based on aperture field formulation and of Auda et al. for thin iris in a rectangular waveguide is based on obstacle current method. In view of the potential application of the above structure for microwave systems, it has been felt desirable to present a method of analysis which is free from these limitations.

In the present work attention has been paid to evaluate the electrical characteristics of thick double apertures in a rectangular waveguide. The analysis is carried out using moment method and aperture field formulation. The aperture field method is used in lieu of obstacle current method because of the following advantages. The application of aperture field method permits use of entire domain sinusoidal basis function which gives a faster convergence than the subsectional basis function used in obstacle current method. The application of Galerkin's technique leads to a symmetric moment matrix which reduces the computation time appreciably. Same formulation can be applied to both inductive as well as capacitive obstacles.

The rectangular aperture with finite axial thickness has been represented as a short rectangular waveguide. The axial thickness is accounted for by introducing higher order waveguide modes in the short waveguide connecting the input and output region [5]. The expressions for the magnetic field generated due to the aperture fields in the two interfaces are derived using modal expansion method [6, Sect. 4.9]. The coupled integral equations resulting from the boundary condition of the magnetic field at the four interfaces are transformed into matrix equation using Galerkin's method. The comparisons between the theoretical results is presented. Theoretical and experimental data are also determined for a single aperture for the sake of comparison with those presented by Marcuvitz.

II. Analysis

Fig. 1(a) shows the cross-sectional view of a rectangular waveguide containing two apertures. The longitudinal-sectional view of the same is shown in Fig. 1(b).

For the purpose of analysis the apertures are considered as sections of waveguides as shown in Fig. 1(b). The modes existing in the two apertures are assumed to be of the type $TE_{0i}(i = p, q)$ [5].

Using modal expansion formulation suggested by Harrington [6, Sect. 4.9], the expressions for the back scattered ($z \leq -t/2$) and forward scattered ($z \geq t/2$) magnetic field at $z = -t/2$ and $z = t/2$ are expressed as

$$H_i(e_p) = V_n\,[\text{sinc}\,\{R_{np}(w_1)\}\cos\{S_{np}(c_1)\} - \text{sinc}\,\{T_{np}(w_1)\}\cos\{U_{np}(c_1)\}]\sin\left(\frac{\pi y}{b}\right) \tag{1}$$

$$H_i(e_q) = V_n\,[\text{sinc}\,\{R_{nq}(w_2)\}\cos\{S_{nq}(c_2)\} - \text{sinc}\,\{T_{nq}(w_2)\}\cos\{U_{nq}(c_2)\}]\sin\left(\frac{\pi y}{b}\right) \tag{2}$$

$$H_o(e_p) = -H_i(e_p) \quad \text{and} \quad H_o(e_q) = -H_i(e_q) \tag{3}$$

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The authors are with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur, Pin-721302, India.
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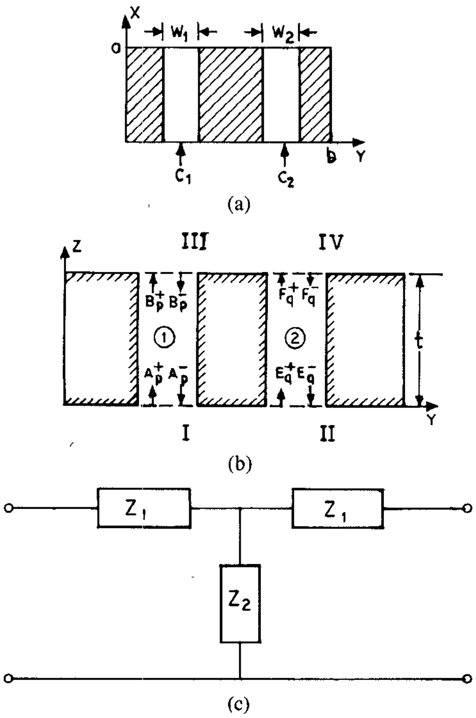

Fig. 1. (a) Cross-sectional view of the two thick apertures in a rectangular waveguide. (b) Longitudinal-sectional view of the same structure. (c) Equivalent T network represented at the plane of symmetry.

where

$$R_{np}(w) = \{\pi w(n/b - p/w)/2\}$$
$$S_{np}(c) = \{n\pi c/b - p\pi/2\}$$
$$T_{np}(w) = \{\pi w(n/b + p/w)/2\}$$
$$U_{np}(c) = \{n\pi c/b + p\pi/2\}. \tag{4}$$

Considering the incident and reflected waves in the waveguide sections having dimensions those of the apertures (designated as a stub waveguide), the boundary conditions at the different interfaces are given by

$$\sum_{p=1}^{M} A_p^+[Y_p e_p - H_{ip}(e_p)] = \sum_{p=1}^{M} A_p^-[Y_p e_p + H_{ip}(e_p)] + \sum_{q=1}^{N} E_q^+ H_{iq}(e_q) + \sum_{q=1}^{N} E_q^- H_{iq}(e_q) + 2H_y^{\text{inc}} \text{ (at interface I)} \tag{5}$$

$$\sum_{q=1}^{N} E_q^+[Y_q e_q - H_{iq}(e_q)] = \sum_{q=1}^{N} E_q^-[Y_q e_q + H_{iq}(e_q)] + \sum_{p=1}^{M} A_p^+ H_{ip}(e_p) + \sum_{p=1}^{M} A_p^- H_{ip}(e_p) + 2H_y^{\text{inc}} \text{ (at interface II)} \tag{6}$$

$$\sum_{p=1}^{M} B_p^+ [Y_p e_p - H_{op}(e_p)] = \sum_{p=1}^{M} B_p^- [Y_p e_p + H_{op}(e_p)] + \sum_{q=1}^{N} F_q^+ H_{oq}(e_q) + \sum_{q=1}^{N} F_q^- H_{oq}(e_q) \text{ (at interface III)} \tag{7}$$

$$\sum_{q=1}^{N} F_q^+[Y_q e_q - H_{oq}(e_q)] = \sum_{q=1}^{N} F_q^-[Y_q e_q + H_{oq}(e_q)] + \sum_{p=1}^{M} B_p^+ H_{op}(e_p) + \sum_{p=1}^{M} B_p^- H_{op}(e_p) \text{ (at interface IV).} \tag{8}$$

In the above equations, the basis functions e_p and e_q are given by

$$e_p = \begin{cases} \sin \dfrac{p\pi}{w_1}\left(y - c_1 + \dfrac{w_1}{2}\right) & \text{for } c_1 - \dfrac{w_1}{2} \le y \le c_1 + \dfrac{w_1}{2} \\ 0, & \text{elsewhere,} \end{cases} \tag{9}$$

in the first stub waveguide, and $0 \le x \le a$,

$$e_q = \begin{cases} \sin \dfrac{q\pi}{w_2}\left(y - c_2 + \dfrac{w_2}{2}\right), & \text{for } c_2 - \dfrac{w_2}{2} \le y \le c_2 + \dfrac{w_2}{2}, \\ 0, & \text{elsewhere,} \end{cases} \tag{10}$$

in the second stub waveguide and $0 \le x \le a$.

Y_p and Y_q are the modal admittances of TE_{0i} ($i = p, q$) modes in the first and second stub waveguide, respectively.

The column matrices (B^+, A^+), (A^-, B^-), (F^+, E^+) and E^-, F^-) representing the forward and backward waves in first and second stub waveguide are related by diagonal matrices whose diagonal elements are given by [5]

$$\theta_{pp} = e^{-\gamma_{0p}t}$$

and

$$\phi_{qq} = e^{-\gamma_{0q}t}$$

at first and second stub waveguide, respectively.

Using these relations and taking the inner product of (5) and (7) with e_r (obtained on replacing p by r) and (6) and (8) with e_s (obtained on replacing q by s), following matrix equations relating the above column matrices are obtained:

$$[L_1^i][A^+] + [L_2^i][A^-] + [L_3^i][E^+] + [L_3^i][E^-] = [G_1^{\text{inc}}] \tag{11}$$

$$[L_4^i][A^+] + [L_4^i][A^-] + [L_5^i][E^+] + [L_6^i][E^-] = [G_2^{\text{inc}}] \tag{12}$$

$$[L_1^o][A^+] + [L_2^o][A^-] + [L_3^o][E^+] + [L_4^o][E^-] = [0] \tag{13}$$

$$[L_5^o][A^+] + [L_6^o][A^-] + [L_7^o][E^+] + [L_8^o][E^-] = [0]. \tag{14}$$

The expressions for the elements of the column matrix G_1^{inc} and G_2^{inc} as well as other matrices $[L_j^i]$ ($j = 1 \cdots 6$) and $[L_j^o]$ ($j = 1 \cdots 8$) are obtained from the inner products mentioned above.

The matrix elements contained the propagation constants (γ_{0p}, $D\gamma_{0q}$) as well as characteristic admittances (Y_p, Y_q) for TE_{0i}($i = p, q$) modes inside the stub waveguides, the expressions for which are given by

$$\gamma_{0p} = \begin{cases} j\sqrt{k^2 - \left(\dfrac{p\pi}{w_1}\right)^2}, & \text{for } k \ge \dfrac{p\pi}{w_1}, \\ \sqrt{\left(\dfrac{p\pi}{w_1}\right)^2 - k^2}, & \text{for } k \le \dfrac{p\pi}{w_1}. \end{cases} \tag{15}$$

$$\gamma_{0q} = \begin{cases} j\sqrt{k^2 - \left(\frac{q\pi}{w_2}\right)^2}, & \text{for } k \geq \frac{q\pi}{w_2}, \\ \sqrt{\left(\frac{q\pi}{w_2}\right)^2 - k^2}, & \text{for } k \leq \frac{q\pi}{w_2}, \end{cases} \tag{16}$$

$$Y_p = \frac{\gamma_{0p}}{j\omega\mu}, \tag{17}$$

$$Y_q = \frac{\gamma_{0q}}{j\omega\mu}. \tag{18}$$

Substituting $n = 1$ in (1)–(4), the expressions for the dominant mode back scattered ($z \leq -t/2$) (H_t^{01}) and forward scattered ($z \geq t/2$)(H_o^{01}) magnetic field are obtained in terms of the known complex amplitude coefficients. From the known value of the back-scattered and forward scattered magnetic field the numerical value of reflection and transmission coefficient is evaluated using the following relations

$$\Gamma = -1.0 + \frac{H_t^{01}}{H^{\text{inc}}} \tag{19}$$

$$T = \frac{H_o^{01}}{H^{\text{inc}}}. \tag{20}$$

The elements of the equivalent lumped network represented at the plane of symmetry, as shown in Fig. 1(c), are given in terms of Γ and T as [7, Sec. 4.7]

$$z_1 = \frac{1 + \Gamma - T}{1 - \Gamma + T} \tag{21}$$

$$z_2 = \frac{2T}{(1 - \Gamma + T)(1 - \Gamma - T)}. \tag{22}$$

In the case of a single aperture, one of the stub waveguide disappears and the number of matrix equation reduces to two instead of four [5]. Evaluation of Γ, T, z_1, and z_2 follows accordingly.

III. Numerical and Experimental Results

Using (1)–(14) the complex amplitude coefficients are evaluated over the frequency range 8.0 to 11.0 GHz at interfaces I, II, III and IV for aperture parameters $w_1 = 0.746$ cm, $w_2 = 1.210$ cm, $c_1 = 0.373$ cm, $c_2 = 1.681$ cm, $t = 0.32$ cm and 0.638 cm and waveguide dimensions $a = 1.016$ cm and $b = 2.286$ cm. The results are found to converge for the number of basis functions $M = N = 5$ and waveguide modes ranging to $m = 0$ and $n = 50$. Substituting the computed values of the amplitude coefficients into (19)–(20) the complex reflection and transmission coefficients are evaluated. Marcuvitz presented theoretical results for the equivalent T network of a centered as well as offset post with rectangular cross-section. Such a structure can be regarded as a combination of two thick apertures. For a post with rectangular cross-sectional dimension $d' = 0.33$ cm and $d'' = 0.32$ cm and 0.638 cm and axis located at a distance of 0.911 cm from the waveguide wall, the corresponding aperture parameters are same as those mentioned above. The theoretical data on the magnitude of the transmission coefficient evaluated using the present method are compared with the data calculated using the equivalent circuit elements given by Marcuvitz. The data evaluated by both methods are presented in Fig. 2(a) for $t = 0.32$ cm and in Fig. 2(b) for $t = 0.638$ cm together with the corresponding experimental data obtained using HP Network Analyzer setup for the sake of comparison.

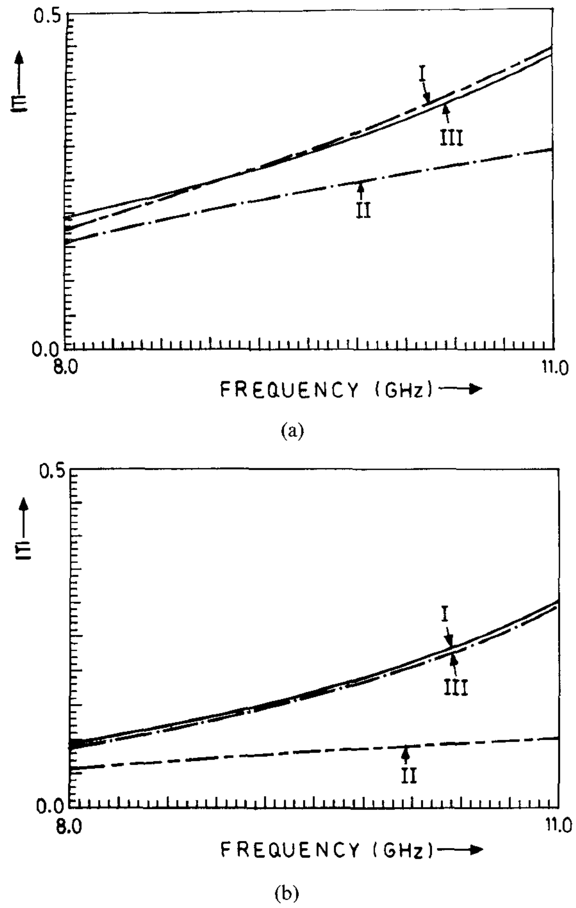

Fig. 2. Variation of the magnitude of transmission coefficient of two thick apertures in a rectangular waveguide with frequency. (a) For aperture thickness of 0.32 cm. i → Present method. ii → Result obtained using Marcuvitz's formula. iii → Experimental curve. (b) Same curves for aperture thickness of 0.638 cm.

Computations on the magnitude of transmission coefficient are carried out for a single aperture $w_1 = 1.0$ cm, $w_2 = 0.0$, $c_1 = 1.3$ cm and $t = 0.32$ cm and 0.638 cm over the frequency range 8.0 to 11.0 GHz using the present method. The theoretical data together with the corresponding experimental data are presented on Fig. 3(a) for $t = 0.32$ cm and Fig. 3(b) for $t = 0.638$ cm for the sake of comparison.

Using (21) and (22) the equivalent T network parameters are evaluated for $w_1 = 1.1$ cm, $w_2 = 1.1$ cm, $c_1 = 0.55$ cm, $c_2 = 1.736$ cm and $t = 0.05$ cm over the frequency range 8.0 to 11.0 GHz. These aperture data correspond to a centered post of rectangular cross-section having $d' = 0.086$ cm and $d'' = 0.05$ cm. The equivalent network parameters are therefore evaluated using the formulation developed by Marcuvitz. The numerical results obtained following the present method as well as those of Marcuvitz are presented in Fig. 4(a) and Fig. 4(b) for the sake of comparison.

IV. Discussion

The results presented in Fig. 2(a), and (b), 3(a) and (b) reveal that there is a good agreement between theoretical results evaluated by present method and experimental results. This agreement justifies the validity of the analysis. It is also found from the data presented in Fig. 2(a) and (b) that the results obtained following

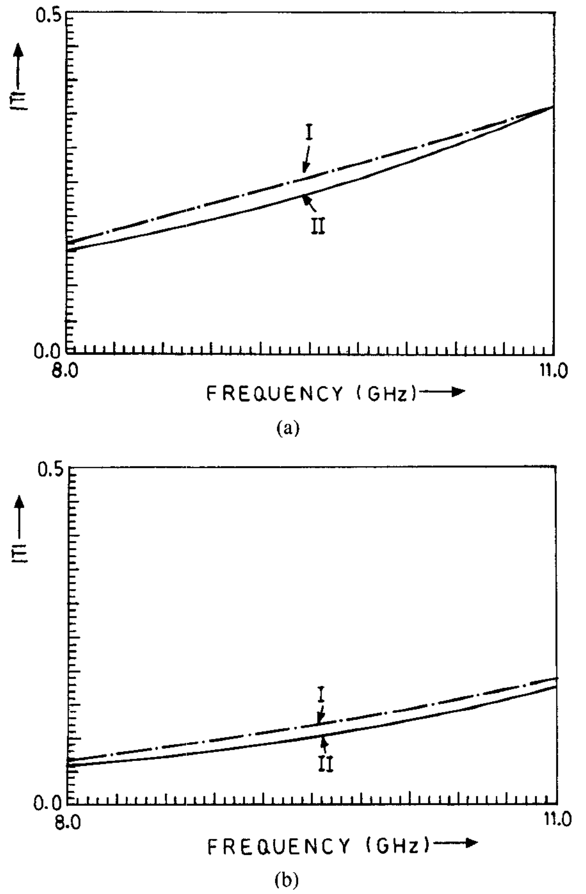

Fig. 3. Variation of the magnitude of transmission coefficient of a single thick aperture in a rectangular waveguide with frequency. (a) For aperture thickness of 0.32 cm. i → Present method. ii → Experimental curve. (b) Same curves for aperture thickness of 0.638 cm.

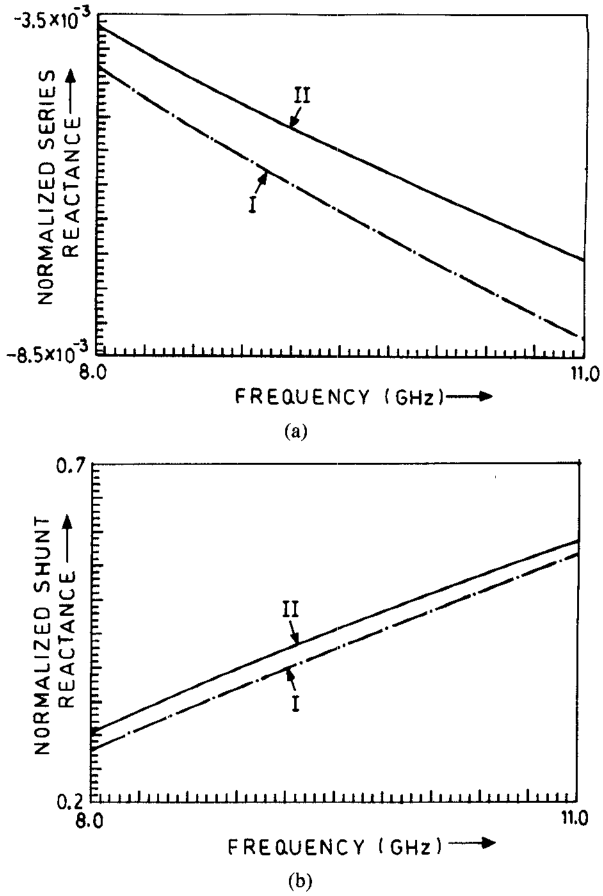

Fig. 4. Variation of the normalized equivalent circuit elements for a centered post of rectangular cross-section with frequency. (a) Plot of normalized series element z_1. i → Present method. ii → Result obtained using Marcuvitz's formula. (b) Same curve for normalized shunt element z_2.

the formulation suggested by Marcuvitz shows a considerable deviation which increases with increasing frequency. The same trend is also observed for the equivalent network parameters presented in Fig. 4(a) ,and (b). It is therefore concluded that the moment method formulation used in the present work yields more accurate results as compared to the variational formulation. This may be ascribed to the inclusion of the effect of the higher order modes in the stub waveguide which the variational method does not permit. It is worthwhile to pointout that the moment method formulation used in the present paper takes into account the mutual interactions between the interfaces I and II as well as III and IV. The method can also be extended to the case of multiple apertures.

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Complex Images of an Electric Dipole in Homogeneous and Layered Dielectrics Between Two Ground Planes

J. J. Yang, Y. L. Chow, G. E. Howard, and D. G. Fang

***Abstract*—In this paper, simple closed form expressions are derived for the vector and scalar potentials of a horizontal electric dipole in**

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J. J. Yang, Y. L. Chow, and G. E. Howard are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3GL.

D. G. Fang is with the East China Institute of Technology, Nanjing, China 210014.

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